4. The Simplex Method

a. Simplex Method - algebraic algorithm for iteratively solving LP problems. Works by starting at a convenient feasible extreme point such as the origin and moving to adjacent extreme points making successive improvements to the value of the objective function.

b. Simplex Tableau - tabular display of the standard form of an LP problem. For the Television Problem (ignoring the fourth constraint for simplification):

maximize $\pi = 3.0x_1 + 8.0x_2$

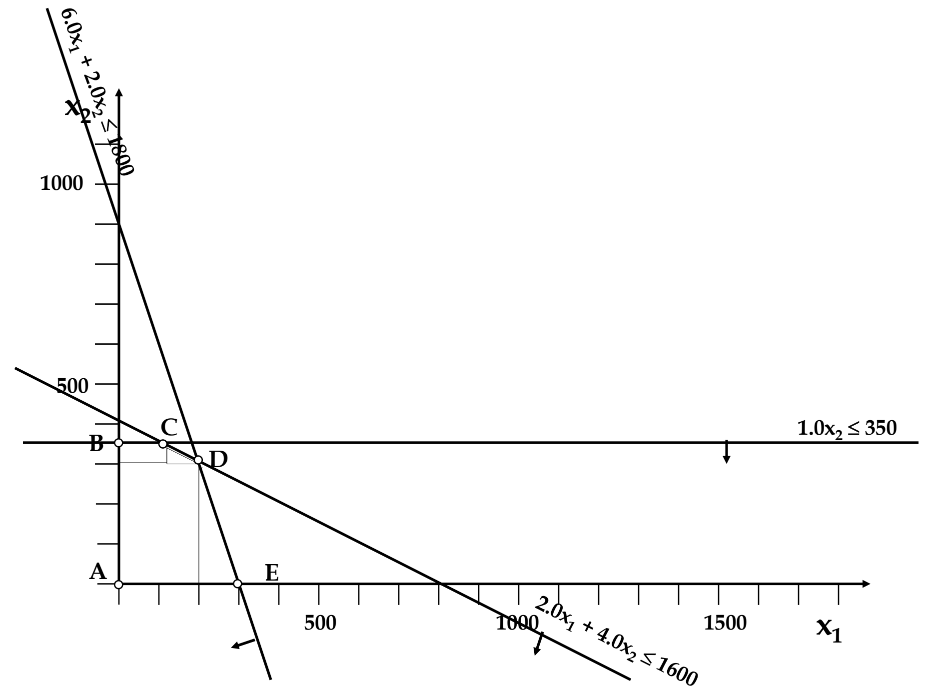
subject to:

$$2.0x_1 + 4.0x_2 + S_A = 1600$$
$$6.0x_1 + 2.0x_2 + S_F = 1800$$
$$1.0x_2 + S_T = 350$$
$$x_1,\ x_2,\ S_A,\ S_F,\ S_T \geq 0$$

The Simplex Tableau would look like this:

Unit Profit		3	8	0	0	0	
	Basic Mix	X_1	X_2	S_A	S_F	S_T	**Solution (π)**
0	S_A	2.0	4.0	1.0	0.0	0.0	1600
0	S_F	6.0	2.0	0.0	1.0	0.0	1800
0	S_T	0.0	1.0	0.0	0.0	1.0	350
	Sac.						
	Imp.						

Note that the variables in the Basic Mix imply that S_A = 1600, S_F = 1800, and S_T = 350 while the nonbasic variables X_1 and X_2 are both equal to zero (i.e., we are at the ORIGIN, i.e., Extreme Point A).



- **Basic Variables - variables that have arbitrarily set nonzero values in the current solution.**
- **Unit Profits - contribution made to the objective function by the variable associated with a particular column.**
- **Exchange Coefficients - amount of the corresponding basic variable that must be given up to produce one unit of the variable in the corresponding column.**
- **Remaining Right Hand Side - difference between the original right-had side and the left hand side for the current solution of the constraint in the corresponding row.**
- **Current π - value of the objective function at the current solution.**

- Unit Sacrifice - per unit loss induced by including one unit of the variable in the corresponding column in the current solution. This is calculated by taking the vector (dot) product of the Unit Profit column vector and the vector comprised by the corresponding Exchange Coefficients column and are often called z_j's.

  For example, the Unit Sacrifice for X_1 is found by taking the dot product of (0 0 0) and (2 6 0), which is 0*2 + 0*6 + 0*0 = 0.0.

- Unit Improvement - net per unit change induced by including one unit of the variable in the corresponding column in the current solution. This is calculated by subtracting the Unit Sacrifice from the Unit Profit for a particular column.

  For example, the Unit Improvement for X_1 is found by subtracting 0.0 (the Unit Sacrifice) from 3.0 (the Unit Profit), i.e., $c_1 - z_1$.

Now the Simplex Tableau looks like this:

Unit Profit		3	8	0	0	0	
	Basic Mix	X_1	X_2	S_A	S_F	S_T	Solution (π)
0	S_A	2.0	4.0	1.0	0.0	0.0	1600
0	S_F	6.0	2.0	0.0	1.0	0.0	1800
0	S_T	0.0	1.0	0.0	0.0	1.0	350
	Sac.	0	0	0	0	0	0
	Imp.	3	8	0	0	0	----

Note that introducing either nonbasic variable (X_1 or X_2) could increase the value of the objective function at this point.

Let's look at the components of the Simplex Tableau

Unit Profits (Objective Function Coefficients or c_j's)

Unit Profit		3	8	0	0	0	
	Basic Mix	X_1	X_2	S_A	S_F	S_T	**Solution** (π)
0	S_A	2.0	4.0	1.0	0.0	0.0	1600
0	S_F	6.0	2.0	0.0	1.0	0.0	1800
0	S_T	0.0	1.0	0.0	0.0	1.0	350
	Sac.	0	0	0	0	0	0
	Imp.	3	8	0	0	0	----

Exchange Coefficients

Basic Variables

Unit Improvement Row

Unit Sacrifice Row

Current π

Remaining Right-Hand Side Values

c. Steps of the Simplex Method

i) Formulate the problem in Standard Form, then construct the initial tableau from this formulation.

ii) Find the initial Sacrifice and Improvement rows.

iii) Apply the Entry Criterion - Find the current nonbasic variable that will improve the objective function at the greatest rate. This variable is the entering variable.

Circle the column corresponding to this variable. This is referred to as the *Pivot Column.*

Note that if no improvement can be found, the solution represented by the current tableau is optimal.

iv) Apply the Exit Criterion - create exchange ratios for each variable that is currently basic by dividing the Solution value by the Exchange Coefficient for the corresponding row.

The *smallest nonnegative* Exchange Ratio corresponds to the Exiting Variable.

Circle the row corresponding to this variable. This is referred to as the *Pivot Row*.

If two (or more) basic variables are tied for the smallest nonnegative Exchange Ratio, break the tie arbitrarily.

v) Construct a new (revised) simplex tableau

Replace the basic mix label of the exiting variable with that of the entering variable (leaving all other basic mix labels undisturbed).

Change the Unit Profit (or Cost) column value of the exiting variable to that of the entering variable (leaving all other basic mix labels undisturbed).

Use Pivot Operations to recompute row values and obtain a new (revised) set of Exchange Coefficients.

This represents completion of one Iteration of the simplex algorithm.

vi) Return to Step 2.

Pivot Element - intersection of the Pivot Row and Pivot Column.

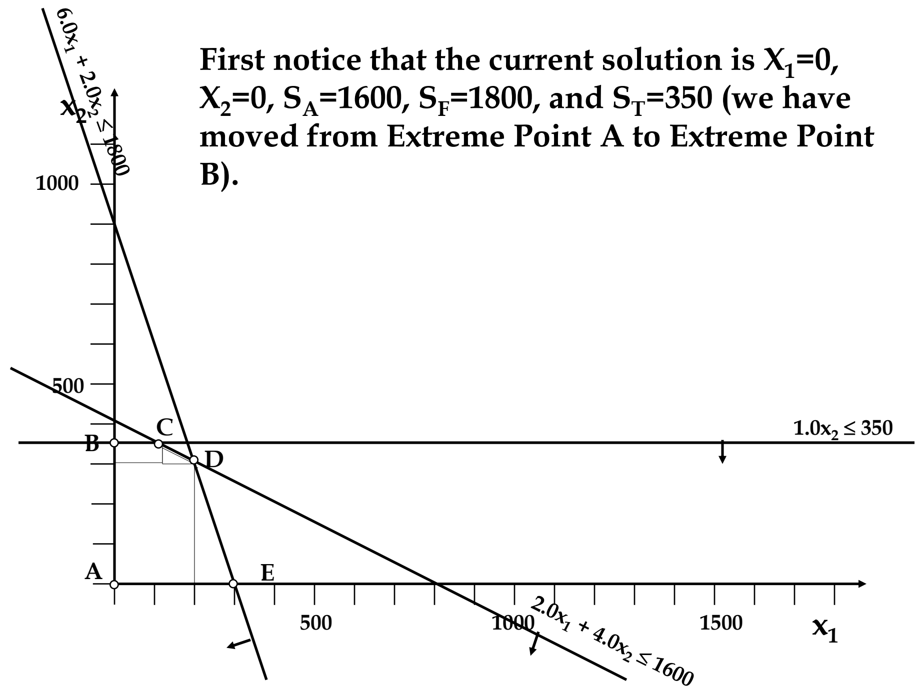
Pivot Operations - linear algebra used to convert the current Pivot Element to a value of 1 and all other elements in the Pivot Column to values of 0.

Let's look at solving the Television Problem using Simplex.

The original tableau (with the initial Sacrifice and Improvement rows) looks like this:

Unit Profit		3	8	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	Solution (π)	Exchange Ratios
0	S_A	2.0	4.0	1.0	0.0	0.0	1600	
0	S_F	6.0	2.0	0.0	1.0	0.0	1800	
0	S_T	0.0	1.0	0.0	0.0	1.0	350	
	Sac.	0	0	0	0	0	0	
	Imp.	3	8	0	0	0	----	

The current nonbasic variable that will improve the objective function at the greatest rate is X_2 . This variable is the entering variable.



First notice that the current solution is X_1=0, X_2=0, S_A=1600, S_F=1800, and S_T=350 (we have moved from Extreme Point A to Extreme Point B).

Now that we have circled the column corresponding to this variable (the *Pivot Column*) we must identify the Exiting Variable

Unit Profit		3	8	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	Solution (π)	Exchange Ratios
0	S_A	2.0	4.0	1.0	0.0	0.0	1600	1600/4
0	S_F	6.0	2.0	0.0	1.0	0.0	1800	1800/2
0	S_T	0.0	1.0	0.0	0.0	1.0	350	350/1
	Sac.	0	0	0	0	0	0	
	Imp.	3	8	0	0	0	----	

The current basic variable with the *smallest nonnegative* Exchange Ratio is S_T. This is the Exiting Variable.

Now that we have circled the row corresponding to this variable (the *Pivot Row*) we can commence with the Pivot Operations.

We first want to perform some linear algebra that will result in the current Pivot Element taking a value of 1 and all other elements in the Pivot Column to values of 0.

$r_3' = 1.0r_3 = (0.0 \quad \textcircled{1.0} \quad 0.0 \quad 0.0 \quad 1.0 \quad 350)$

We are very fortunate - the Exchange Coefficient for the pivot value is already 1, so we do not have to do anything.

Note that the revised row 3 (r_3') will be substituted for the original row 3 (r_3) in the next tableau.

Now we want to perform some linear algebra that will result in all other elements in the current Pivot Column taking on values of 0.

Let's start with row 1 (r_1). Remember, we want to perform some linear algebra that will result in the element in this row and the current Pivot Column taking a value of 0.

We could subtract $4*r_3'$ from r_1.

r_1 =	(2.0	4.0	1.0	0.0	0.0	1600)
$-4r_3'$ = -4	(0.0	1.0	0.0	0.0	1.0	350)
r_1' =	(2.0	0.0	1.0	0.0	-4.0	200)

The revised row 1 (r_1') will be substituted for the original row 1 (r_1) in the next tableau.

Now we focus on row 2 (r_2). Again, we want to perform some linear algebra that will result in the element in this row and the current Pivot Column taking a value of 0.

We could subtract $2*r_3'$ from r_2.

$$
\begin{array}{rl}
r_2 = & (6.0 \quad 2.0 \quad 0.0 \quad 1.0 \quad 0.0 \quad 1800) \\
-2r_3' = -2 & (0.0 \quad 1.0 \quad 0.0 \quad 0.0 \quad 1.0 \quad 350) \\
\hline
r_2' = & (6.0 \quad 0.0 \quad 0.0 \quad 1.0 \quad -2.0 \quad 1100)
\end{array}
$$

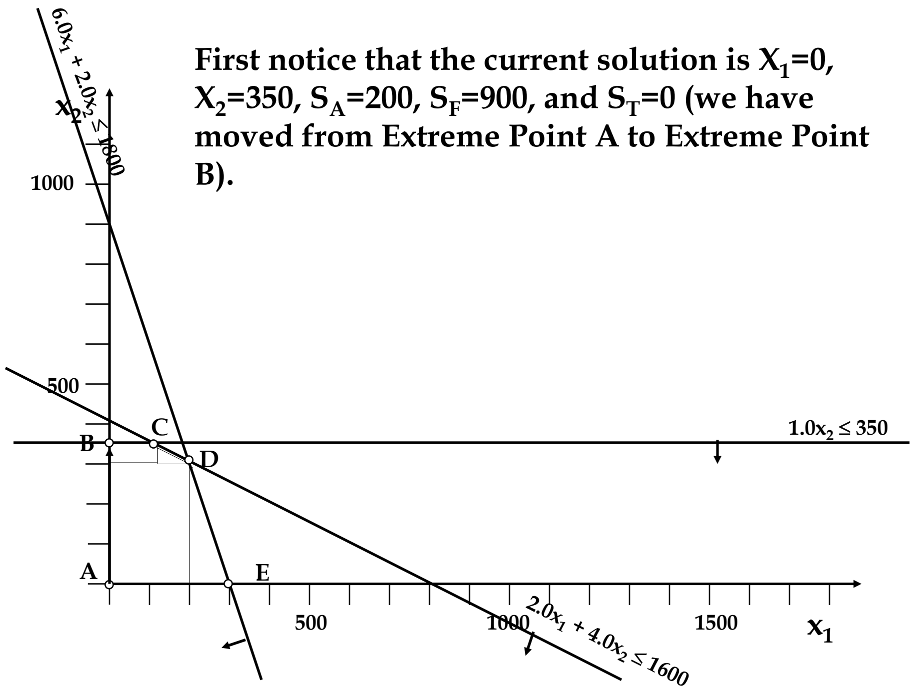
The revised row 2 (r_2') will be substituted for the original row 2 (r_2) in the next tableau.

Since we have updated all rows in the tableau, we are ready to create a revised tableau.

First we substitute the revised rows r_1', r_2', and r_3' for the original rows r_1, r_2, and r_3 (don't forget to update to Basic Mix label and Unit Profit column for r_3!).

Unit Profit		3	8	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	Solution (π)	Exchange Ratios
0	S_A	2.0	0.0	1.0	0.0	-4.0	200	
0	S_F	6.0	0.0	0.0	1.0	-2.0	1100	
8	X_2	0.0	1.0	0.0	0.0	1.0	350	
	Sac.							
	Imp.							

Now we recalculate the Unit Sacrifice and Unit Improvement Rows.



First notice that the current solution is X_1=0, X_2=350, S_A=200, S_F=900, and S_T=0 (we have moved from Extreme Point A to Extreme Point B).

Once we substitute the revised Unit Sacrifice and Unit Improvement Rows into the new tableau, we can identify the Entering variable.

Unit Profit		3	8	0	0	0		
	Basic Mix	**X_1**	**X_2**	**S_A**	**S_F**	**S_T**	**Solution (π)**	**Exchange Ratios**
0	S_A	2.0	0.0	1.0	0.0	-4.0	200	
0	S_F	6.0	0.0	0.0	1.0	-2.0	1100	
8	X_2	0.0	1.0	0.0	0.0	1.0	350	
	Sac.	0	8	0	0	8	2800	
	Imp.	3	0	0	0	-8	----	

X_1 will enter the basic solution on the next iteration.

Calculation of the Exchange Ratios leads us to the Exiting variable.

Unit Profit		3	8	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	**Solution (π)**	**Exchange Ratios**
0	S_A	2.0	0.0	1.0	0.0	-4.0	200	200/2
0	S_F	6.0	0.0	0.0	1.0	-2.0	1100	1100/6
8	X_2	0.0	1.0	0.0	0.0	1.0	350	350/0
	Sac.	0	8	0	0	8	2800	
	Imp.	3	0	0	0	-8	----	

S_A will exit the basic solution on the next iteration.

We first want to perform some linear algebra that will result in the current Pivot Element taking a value of 1 and all other elements in the Pivot Column to values of 0.

$r_1' = (1/2)r_1 = (1.0 \quad 0.0 \quad 1/2 \quad 0.0 \quad -2.0 \quad 100)$

Note that the revised row 1 (r_1') will be substituted for the original row 1 (r_1) in the next tableau.

Now we want to perform some linear algebra that will result in all other elements in the current Pivot Column taking on values of 0.

Let's start with row 2 (r_2). Again, we want to perform some linear algebra that will result in the element in this row and the current Pivot Column taking a value of 0.

We could subtract 6*r_1' from r_2.

r_2 =	(6.0	0.0	0.0	1.0	-2.0	1100)
$-6r_1'$ = -6(1.0		0.0	1/2	0.0	-2.0	100)
r_2' =	(0.0	0.0	-3.0	1.0	10.0	500)

The revised row 2 (r_2') will be substituted for the original row 2 (r_2) in the next tableau.

Now we focus on row 3 (r_3). Remember, we want to perform some linear algebra that will result in the element in this row and the current Pivot Column taking a value of 0.

We could subtract 0*r_1' from r_3.

r_3 =	(0.0	1.0	0.0	0.0	1.0	350)
$-0r_1'$ = -0(1.0		0.0	1/2	0.0	-2.0	100)
r_3' =	(0.0	1.0	0.0	0.0	1.0	350)

Notice that we really didn't have to do an operation on row 3 (r_3) for this iteration (why?)

The revised row 3 (r_3') will be substituted for the original row 3 (r_3) in the next tableau.

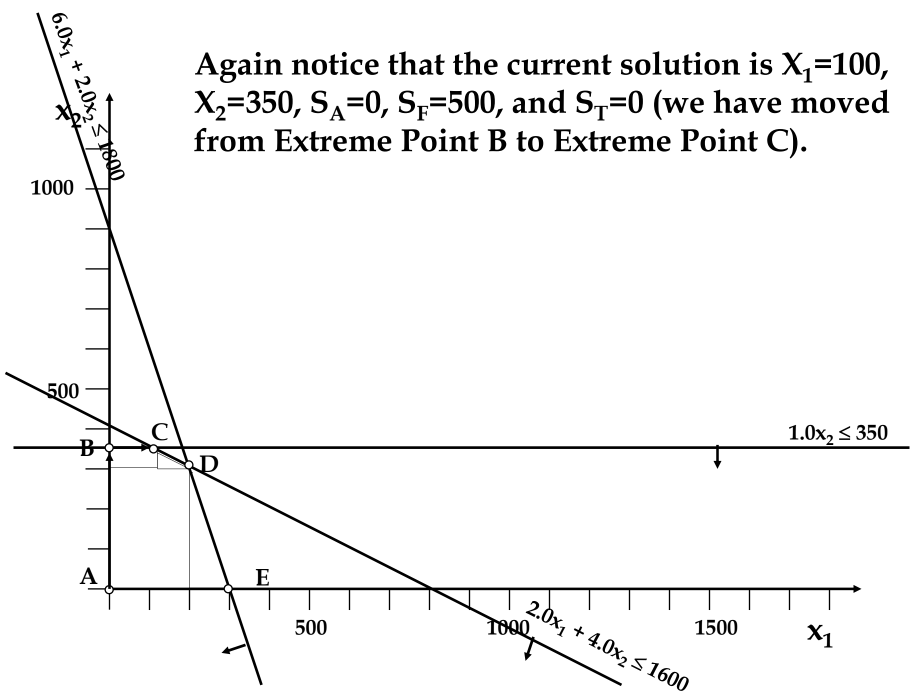
Since we have updated all rows in the tableau, we are ready to create a revised tableau.

First we substitute the revised rows r_1', r_2', and r_3' for the original rows r_1, r_2, and (Don't forget to update to Basic Mix label and Unit Profit column for r_1!).

Unit Profit		3	8	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	Solution (π)	Exchange Ratios
3	X_1	1.0	0.0	1/2	0.0	-2.0	100	
0	S_F	0.0	0.0	-3.0	1.0	10.0	500	
8	X_2	0.0	1.0	0.0	0.0	1.0	350	
	Sac.							
	Imp.							

Now we recalculate the Unit Sacrifice and Unit Improvement Rows.

Again notice that the current solution is X_1=100, X_2=350, S_A=0, S_F=500, and S_T=0 (we have moved from Extreme Point B to Extreme Point C).



Once we substitute the revised Unit Sacrifice and Unit Improvement Rows into the new tableau, we can identify the Entering variable.

Unit Profit		3	8	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	Solution (π)	Exchange Ratios
3	X_1	1.0	0.0	1/2	0.0	-2.0	100	
0	S_F	0.0	0.0	-3.0	1.0	10.0	500	
8	X_2	0.0	1.0	0.0	0.0	1.0	350	
	Sac.	3	8	3/2	0	2	3100	
	Imp.	0	0	-3/2	0	-2	----	

There are no positive Unit Improvements, so we have found the optimal solution in two iterations..

The optimal solution is

X_1= 100
X_2= 350
S_A= 0
S_F= 500
S_T= 0
π = 3100

In other words, produce 100 Black & White Sets and 350 Color Sets at a profit of $3,100. There will be 500 unused fabrication hours and no unused assembly labor hours or color television tubes.

Interpreting the Intermediate and Final Simplex Tableaus

First we'll look at the Exchange Coefficients

Unit Profit		3	8	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	Solution (π)	Exchange Ratios
3	X_1	1.0	0.0	1/2	0.0	-2.0	100	
0	S_F	0.0	0.0	-3.0	1.0	10.0	500	
8	X_2	0.0	1.0	0.0	0.0	1.0	350	
	Sac.	3	8	3/2	0	2	3100	
	Imp.	0	0	-3/2	0	-2	----	

Consider the entries in the S_A column. The first entry (1/2) indicates that increasing the value of S_A by one unit in the current solution would cost us 1/2 of an X_1 (i.e, result in 0.5 decrease in the solution value of X_1).

Similarly, the second entry in the S_A column (-3.0) indicates that increasing the value of S_A by one unit in the current solution would cost us -3.0 of an S_F (i.e, result in 3.0 increase in the solution value of S_F).

Unit Profit		3	8	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	Solution (π)	Exchange Ratios
3	X_1	1.0	0.0	1/2	0.0	-2.0	100	
0	S_F	0.0	0.0	-3.0	1.0	10.0	500	
8	X_2	0.0	1.0	0.0	0.0	1.0	350	
	Sac.	3	8	3/2	0	2	3100	
	Imp.	0	0	-3/2	0	-2	----	

The third entry in the S_A column (0.0) indicates that increasing the value of S_A by one unit in the current solution would cost us 0.0 of an X_2 (why?).

How would we interpret the Exchange Coefficients in the X_1 column of the final simplex tableau?

Unit Profit		3	8	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	**Solution (π)**	**Exchange Ratios**
3	X_1	1.0	0.0	1/2	0.0	-2.0	100	
0	S_F	0.0	0.0	-3.0	1.0	10.0	500	
8	X_2	0.0	1.0	0.0	0.0	1.0	350	
	Sac.	3	8	3/2	0	2	3100	
	Imp.	0	0	-3/2	0	-2	----	

Now consider the values in the Sacrifice Row. The sacrifice for S_A is -3/2, indicating that increasing the value of S_A by one unit (holding back an additional assembly labor hour) at this point would reduce the objective function's value by 3/2.

Unit Profit		3	8	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	**Solution (π)**	**Exchange Ratios**
3	X_1	1.0	0.0	1/2	0.0	-2.0	100	
0	S_F	0.0	0.0	-3.0	1.0	10.0	500	
8	X_2	0.0	1.0	0.0	0.0	1.0	350	
	Sac.	3	8	3/2	0	2	3100	
	Imp.	0	0	-3/2	0	-2	----	

Since an additional unit of S_A contributes 0 to the objective function, the net change would be -3/2.

Similarly, the sacrifice for S_T indicate that increasing the value of S_T by one unit at this point would reduce the objective function's value by 2.

Unit Profit		3	8	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	Solution (π)	Exchange Ratios
3	X_1	1.0	0.0	1/2	0.0	-2.0	100	
0	S_F	0.0	0.0	-3.0	1.0	10.0	500	
8	X_2	0.0	1.0	0.0	0.0	1.0	350	
	Sac.	3	8	3/2	0	2	3100	
	Imp.	0	0	-3/2	0	-2	----	

What do the sacrifice values for the basic variables (X_1, X_2, and S_F) suggest?

Comments on the Simplex Method

Math errors commonly occur when executing the Simplex Algorithm by hand. Keep these considerations in mind to minimize your errors:

- Work in fractions, not decimals
- Values in the Solution Column can never be negative
- Basic Variable Columns should consist of a 1 where the row and column corresponding to the basic variable intersect and zeros in all other positions of the column corresponding to the basic variable (with the possible exception of the sacrifice row)
- The value of the objective function in the updated tableau should be no worse than the objective function in the previous tableau
- No constraints should be violated at any time

d. Surplus Variables and Artificial Variables

Recall our original Television Problem formulation (with the minimum Color Television Sets constraint restored):

maximize $\pi = 3.0x_1 + 8.0x_2$

subject to:

$$2.0x_1 + 4.0x_2 \le 1600 \text{ (\# of assembly hours available)}$$
$$6.0x_1 + 2.0x_2 \le 1800 \text{ (\# of fabrication hours available)}$$
$$1.0x_2 \le 350 \text{ (\# of available color tubes)}$$
$$1.0x_2 \ge 75 \text{ (minimum \# of color sets)}$$
$$x_1, \quad x_2 \ge 0 \text{ (nonnegativity)}$$

Where x_1 **is the number of black & white sets produced**
x_2 **is the number of color sets produced**

Now also recall our previous definition of a Surplus variable:

Surplus Variable - variable subtracted from the left-hand sides of a greater than or equal to constraint to convert it to an equality.

For example, we can convert the constraint

$$1.0x_2 \ge 75 \text{ (minimum \# of color sets)}$$

to an equality by subtracting a surplus variable S_C **from the left-hand side, i.e.,**

$$1.0x_2 - S_C = 75 \text{ (minimum \# of color sets)}$$

Note that S_C **represents number of color sets produced beyond the minimum number required!**

By using Surplus Variables (as well as Slack Variables) we can put any LP problem into Standard Form:

maximize $\pi = 3.0x_1 + 8.0x_2 + 0S_A + 0S_F + 0S_T + 0S_C$

subject to:

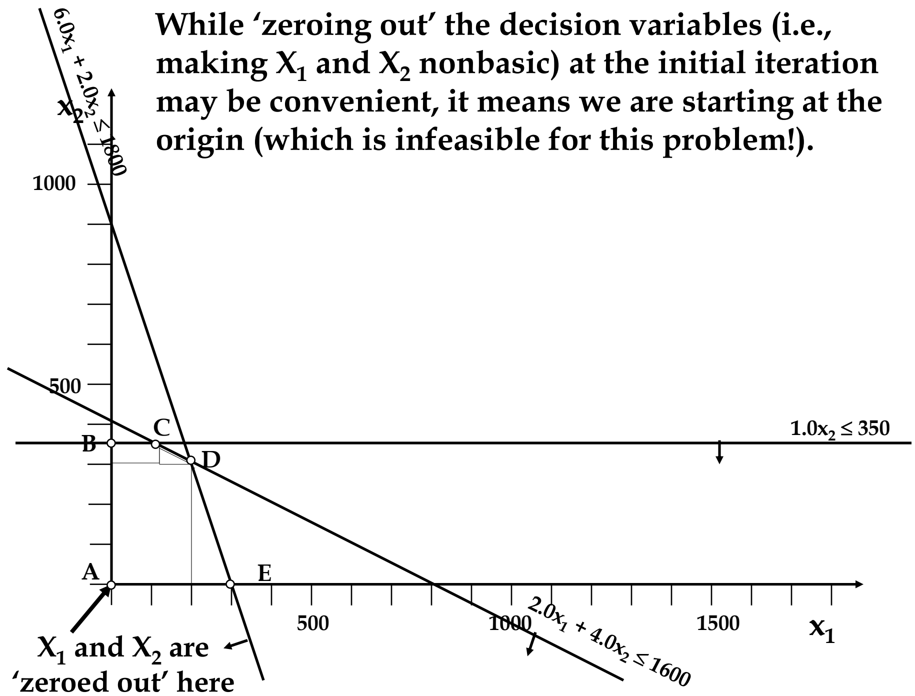
$$
\begin{aligned}
2.0x_1 + 4.0x_2 + S_A &= 1600 \\
6.0x_1 + 2.0x_2 + S_F &= 1800 \\
1.0x_2 + S_T &= 350 \\
1.0x_2 - S_C &= 75 \\
x_1,\ x_2,\ S_A,\ S_F,\ S_T,\ S_C &\geq 0
\end{aligned}
$$

Where x_1 **is the number of black & white sets produced**
x_2 **is the number of color sets produced**
S_A **is the slack assembly labor hours**
S_F **is the slack fabrication labor hours**
S_T **is the slack color tubes**
S_C **is the surplus color tubes beyond demand**

However, notice that the Surplus Variable (S_C) cannot be part of the basic solution - if we make X_2 nonbasic, we end up with S_C = -75 (which violates nonnegativity).

Why is this happening?

While 'zeroing out' the decision variables (i.e., making X_1 and X_2 nonbasic) at the initial iteration may be convenient, it means we are starting at the origin (which is infeasible for this problem!).



This will be true of *any Surplus Variable* (it cannot be in the initial basic mix).

We need to find a way to address this issue.

To do so we need to introduce an *Artificial Variable*.

Artificial Variable - dummy variable added to an equal to (=) or a greater than or equal to (≥) constraint of an LP problem in order to obtain a feasible (nonnegative) initial solution. Often denoted a_i (or more commonly A_i).

- **Artificial Variables have no economic interpretation**
- **an Artificial Variable must have a value of zero at optimality or the LP problem has no solution**

To force an Artificial Variable out of the basic solution, we give it an enormously unattractive objective function coefficient (called the *Big M*)

Incorporating an Artificial Variable into the Television Problem formulation would result in:

$$\text{maximize } \pi = 3.0x_1 + 8.0x_2 + 0S_A + 0S_F + 0S_T + 0S_C - MA_C$$

$$
\begin{aligned}
\text{subject to: } \quad & 2.0x_1 + 4.0x_2 + S_A = 1600 \\
& 6.0x_1 + 2.0x_2 + S_F = 1800 \\
& 1.0x_2 + S_T = 350 \\
& 1.0x_2 - S_C + A_C = 75 \\
& x_1,\ x_2,\ S_A,\ S_F,\ S_T,\ S_C,\ A_C \geq 0
\end{aligned}
$$

Notice that the objective function coefficient for the artificial variable is -M (an arbitrarily large negative value). Since we are maximizing the objective function, this should ensure that the artificial variable is not part of the final optimal solution.

Our initial basic solution can now be:

$$
\begin{aligned}
X_1 &= 0 \\
X_2 &= 0 \\
S_A &= 1600 \\
S_F &= 1800 \\
S_T &= 350 \\
S_C &= 0 \\
A_C &= 75
\end{aligned}
$$

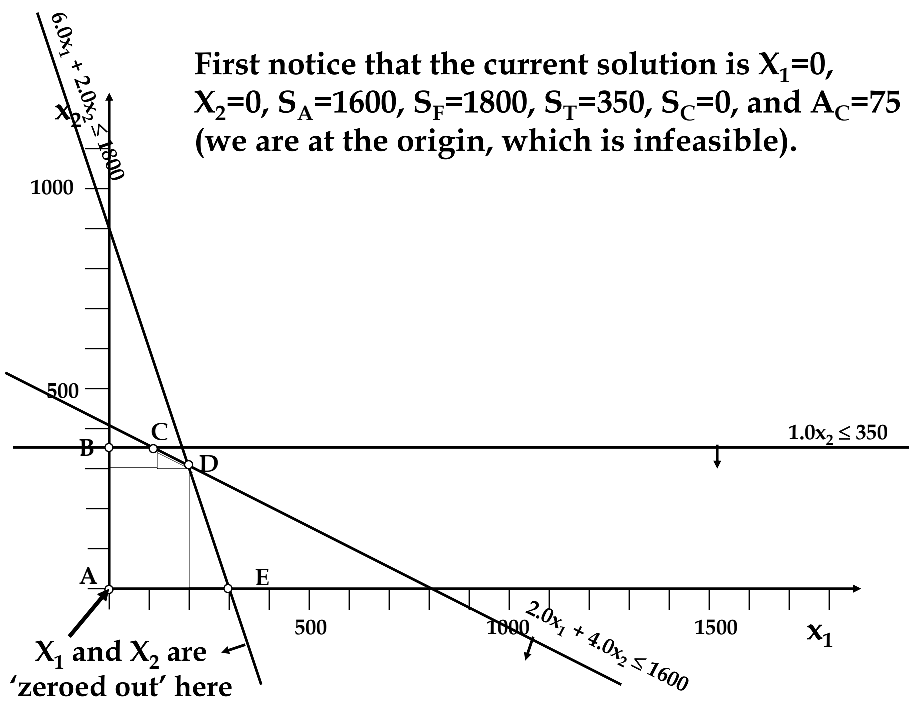
Now let's use simplex to solve the original Television Problem formulation (with the minimum Color Television Sets constraint restored):

The original tableau (with the initial Sacrifice and Improvement rows) looks like this:

Unit Profit		3	8	0	0	0	0	-M		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	S_C	A_C	Solution (π)	Exchange Ratios
0	S_A	2.0	4.0	1.0	0.0	0.0	0.0	0.0	1600	
0	S_F	6.0	2.0	0.0	1.0	0.0	0.0	0.0	1800	
0	S_T	0.0	1.0	0.0	0.0	1.0	0.0	0.0	350	
-M	A_C	0.0	1.0	0.0	0.0	0.0	-1.0	1.0	75	
	Sac.	0	-M	0	0	0	M	-M	-M	
	Imp.	3	M+8	0	0	0	-M	0	----	

The current nonbasic variable that will improve the objective function at the greatest rate is X_2 .

First notice that the current solution is X_1=0, X_2=0, S_A=1600, S_F=1800, S_T=350, S_C=0, and A_C=75 (we are at the origin, which is infeasible).



Now that we have circled the column corresponding to this variable (the *Pivot Column*) we must identify the Exiting Variable

Unit Profit		3	8	0	0	0	0	-M		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	S_C	A_C	Solution (π)	Exchange Ratios
0	S_A	2.0	4.0	1.0	0.0	0.0	0.0	0.0	1600	1600/4
0	S_F	6.0	2.0	0.0	1.0	0.0	0.0	0.0	1800	1800/2
0	S_T	0.0	1.0	0.0	0.0	1.0	0.0	0.0	350	350/1
-M	A_C	0.0	1.0	0.0	0.0	0.0	-1.0	1.0	75	75/1
	Sac.	0	-M	0	0	0	M	-M	-75M	
	Imp.	3	M+8	0	0	0	-M	0	----	

The current basic variable with the *smallest nonnegative* Exchange Ratio is A_C. This is the Exiting Variable.

Now that we have circled the row corresponding to this variable (the *Pivot Row*) we can commence with the Pivot Operations.

We first want to perform some linear algebra that will result in the current Pivot Element taking a value of 1 and all other elements in the Pivot Column to values of 0.

$$r_4' = 1.0 r_4 = (0.0 \quad \textcircled{1.0} \quad 0.0 \quad 0.0 \quad 0.0 \quad -1.0 \quad 1.0 \quad 75)$$

Again we are fortunate - the Exchange Coefficient for the pivot value is already 1, so we do not have to do anything.

The revised row 4 (r_4') will be substituted for the original row 4 (r_4) in the next tableau.

Now we want to perform some linear algebra that will result in all other elements in the current Pivot Column taking on values of 0.

Let's start with row 1 (r_1). Remember, we want to perform some linear algebra that will result in the element in this row and the current Pivot Column taking a value of 0.

We could subtract 4*r_4' from r_1.

r_1 =	(2.0	4.0	1.0	0.0	0.0	0.0	0.0	1600)
$-4r_4'$ = -4	(0.0	1.0	0.0	0.0	0.0	-1.0	1.0	75)
r_1' =	(2.0	0.0	1.0	0.0	0.0	4.0	-4.0	1300)

The revised row 1 (r_1') will be substituted for the original row 1 (r_1) in the next tableau.

Now we focus on row 2 (r_2). Again, we want to perform some linear algebra that will result in the element in this row and the current Pivot Column taking a value of 0.

We could subtract 2*r_4' from r_2.

r_2 =	(6.0	2.0	0.0	1.0	0.0	0.0	0.0	1800)
$-2r_4'$ = -2	(0.0	1.0	0.0	0.0	0.0	-1.0	1.0	75)
r_2' =	(6.0	0.0	0.0	1.0	0.0	2.0	-2.0	1650)

The revised row 2 (r_2') will be substituted for the original row 2 (r_2) in the next tableau.

Finally we focus on row 3 (r_3). Again, we want to perform some linear algebra that will result in the element in this row and the current Pivot Column taking a value of 0.

We could subtract $1*r_4'$ from r_3.

r_3	=	(0.0	1.0	0.0	0.0	1.0	0.0	0.0	350)
$-1r_4'$	= -1	(0.0	1.0	0.0	0.0	0.0	-1.0	1.0	75)
r_3'	=	(0.0	0.0	0.0	0.0	1.0	1.0	-1.0	275)

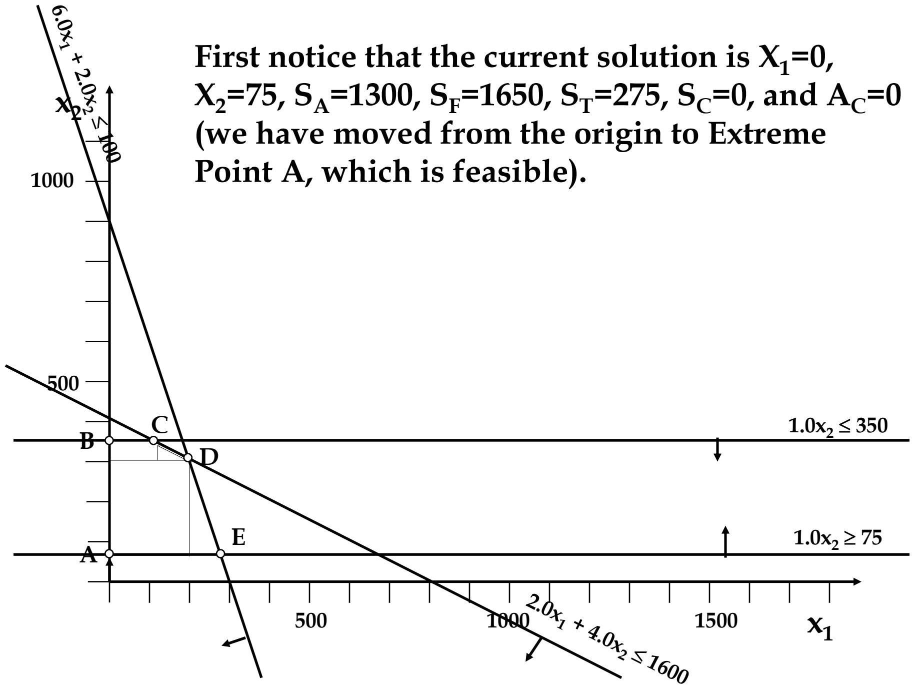
The revised row 3 (r_3') will be substituted for the original row 3 (r_3) in the next tableau.

Since we have updated all rows in the tableau, we are ready to create a revised tableau.

First we substitute the revised rows r_1', r_2', r_3', and r_4' for the original rows r_1, r_2, r_3, and r_4 (don't forget to update to Basic Mix label and Unit Profit column for r_4!).

Unit Profit		3	8	0	0	0	0	-M		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	S_C	A_C	Solution (π)	Exchange Ratios
0	S_A	2.0	0.0	1.0	0.0	0.0	4.0	-4.0	1300	
0	S_F	6.0	0.0	0.0	1.0	0.0	2.0	-2.0	1650	
0	S_T	0.0	0.0	0.0	0.0	1.0	1.0	-1.0	275	
8	X_2	0.0	1.0	0.0	0.0	0.0	-1.0	1.0	75	
	Sac.									
	Imp.									

Now we recalculate the Unit Sacrifice and Unit Improvement Rows.



First notice that the current solution is X_1=0, X_2=75, S_A=1300, S_F=1650, S_T=275, S_C=0, and A_C=0 (we have moved from the origin to Extreme Point A, which is feasible).

At this point we can discard the Artificial Variable column. Once we substitute the revised Unit Sacrifice and Unit Improvement Rows into the new tableau, we can identify the Entering variable.

Unit Profit		3	8	0	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	S_C	**Solution (π)**	**Exchange Ratios**
0	S_A	2.0	0.0	1.0	0.0	0.0	4.0	1300	
0	S_F	6.0	0.0	0.0	1.0	0.0	2.0	1650	
0	S_T	0.0	0.0	0.0	0.0	1.0	1.0	275	
8	X_2	0.0	1.0	0.0	0.0	0.0	-1.0	75	
	Sac.	0	8	0	0	0	-8	600	
	Imp.	3	0	0	0	0	8	----	

S_C will enter the basic solution on the next iteration.

Now that we have circled the column corresponding to this variable (the *Pivot Column*) we must identify the Exiting Variable

Unit Profit		3	8	0	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	S_C	Solution (π)	Exchange Ratios
0	S_A	2.0	0.0	1.0	0.0	0.0	4.0	1300	1300/4
0	S_F	6.0	0.0	0.0	1.0	0.0	2.0	1650	1650/2
0	S_T	0.0	0.0	0.0	0.0	1.0	1.0	275	275/1
8	X_2	0.0	1.0	0.0	0.0	0.0	-1.0	75	75/-1
	Sac.	0	8	0	0	0	-8	600	
	Imp.	3	0	0	0	0	8	----	

The current basic variable with the *smallest nonnegative* Exchange Ratio is S_T. This is the Exiting Variable.

Now that we have circled the row corresponding to this variable (the *Pivot Row*) we can perform the necessary Pivot Operations.

We first want to perform some linear algebra that will result in the current Pivot Element taking a value of 1 and all other elements in the Pivot Column to values of 0.

$$r_3' = 1r_3 = (0.0 \quad 0.0 \quad 0.0 \quad 0.0 \quad 1.0 \quad \boxed{1.0} \quad 275)$$

The revised row 3 (r_3') will be substituted for the original row 3 (r_3) in the next tableau.

We now must perform some linear algebra that will result in all other elements in the current Pivot Column taking on values of 0.

Let's start with row 1 (r_1). Remember, we want to perform some linear algebra that will result in the element in this row and the current Pivot Column taking a value of 0.

We could subtract 4*r_3' from r_1.

r_1 =	(2.0	0.0	1.0	0.0	0.0	4.0	1300)
$-4r_3'$ = -4	(0.0	0.0	0.0	0.0	1.0	1.0	275)
r_1' =	(2.0	0.0	1.0	0.0	-4.0	0.0	200)

The revised row 1 (r_1') will be substituted for the original row 1 (r_1) in the next tableau.

We continue by performing the necessary pivot operations on row 2 (r_2) that will result in the element in this row and the current Pivot Column taking a value of 0.

We could subtract 2*r_3' from r_2.

r_2 =	(6.0	0.0	0.0	1.0	0.0	2.0	1650)
$-2r_3'$ = -2	(0.0	0.0	0.0	0.0	1.0	1.0	275)
r_2' =	(6.0	0.0	0.0	1.0	-2.0	0.0	1100)

The revised row 2 (r_2') will be substituted for the original row 2 (r_2) in the next tableau.

Finally we perform the necessary pivot operations on row 4 (r_4) that will result in the element in this row and the current Pivot Column taking a value of 0.

We could add $1^*r_3'$ to r_4.

r_4 =	(0.0	1.0	0.0	0.0	0.0	-1.0	75)
$+1r_3'$ = +1	(0.0	0.0	0.0	0.0	1.0	1.0	275)
r_4' =	(0.0	1.0	0.0	0.0	1.0	0.0	350)

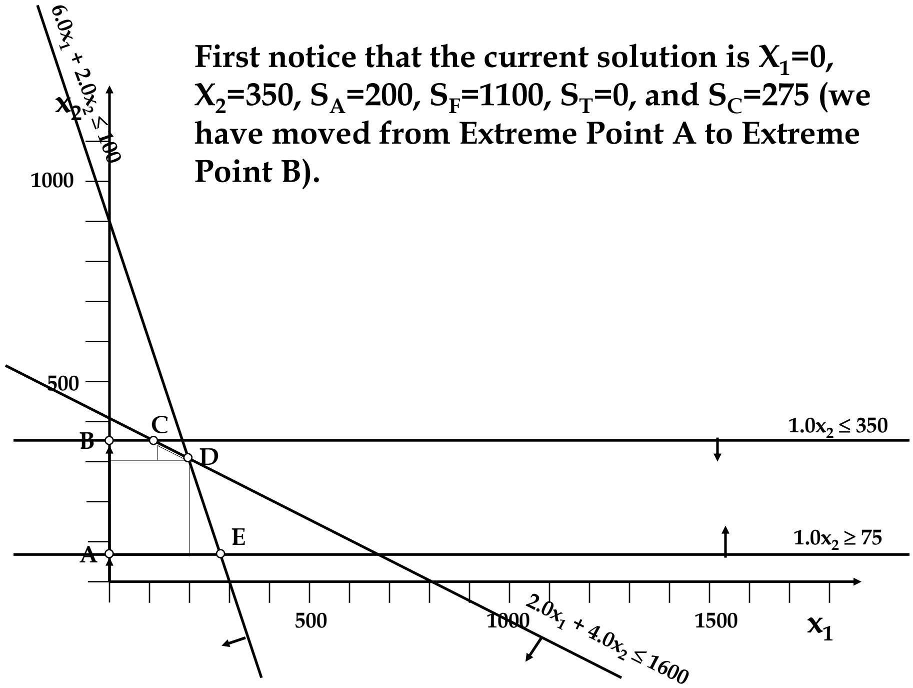
The revised row 4 (r_4') will be substituted for the original row 4 (r_4) in the next tableau.

Since we have updated all rows in the tableau, we are ready to create a revised tableau.

First we substitute the revised rows r_1', r_2', r_3', and r_4' for the original rows r_1, r_2, r_3, and r_4 (don't forget to update to Basic Mix label and Unit Profit column for r_3!).

Unit Profit		3	8	0	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	S_C	Solution (π)	Exchange Ratios
0	S_A	2.0	0.0	1.0	0.0	-4.0	0.0	200	
0	S_F	6.0	0.0	0.0	1.0	-2.0	0.0	1100	
0	S_C	0.0	0.0	0.0	0.0	1.0	1.0	275	
8	X_2	0.0	1.0	0.0	0.0	1.0	0.0	350	
	Sac.								
	Imp.								

Now we recalculate the Unit Sacrifice and Unit Improvement Rows.



First notice that the current solution is X_1=0, X_2=350, S_A=200, S_F=1100, S_T=0, and S_C=275 (we have moved from Extreme Point A to Extreme Point B).

Once we substitute the revised Unit Sacrifice and Unit Improvement Rows into the new tableau, we can identify the Entering variable.

Unit Profit		3	8	0	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	S_C	**Solution (π)**	**Exchange Ratios**
0	S_A	2.0	0.0	1.0	0.0	-4.0	0.0	200	
0	S_F	6.0	0.0	0.0	1.0	-2.0	0.0	1100	
0	S_C	0.0	0.0	0.0	0.0	1.0	1.0	275	
8	X_2	0.0	1.0	0.0	0.0	1.0	0.0	350	
	Sac.	0	8	0	0	8	0	2800	
	Imp.	3	0	0	0	-8	0	----	

X_1 will enter the basic solution on the next iteration.

Now that we have circled the column corresponding to this variable (the *Pivot Column*) we must identify the Exiting Variable

Unit Profit		3	8	0	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	S_C	Solution (π)	Exchange Ratios
0	S_A	2.0	0.0	1.0	0.0	-4.0	0.0	200	200/2
0	S_F	6.0	0.0	0.0	1.0	-2.0	0.0	1100	1100/6
0	S_C	0.0	0.0	0.0	0.0	1.0	1.0	275	275/0
8	X_2	0.0	1.0	0.0	0.0	1.0	0.0	350	350/0
	Sac.	0	8	0	0	8	0	2800	
	Imp.	3	0	0	0	-8	0	----	

The current basic variable with the *smallest nonnegative* Exchange Ratio is S_A. This is the Exiting Variable.

Now that we have circled the row corresponding to this variable (the *Pivot Row*) we can perform the necessary Pivot Operations.

We first want to perform some linear algebra that will result in the current Pivot Element taking a value of 1 and all other elements in the Pivot Column to values of 0.

$r_1' = 1/2 r_1 =$ (1.0 0.0 1/2 0.0 -2.0 0.0 100)

The revised row 1 (r_1') will be substituted for the original row 1 (r_1) in the next tableau.

We now must perform some linear algebra that will result in all other elements in the current Pivot Column taking on values of 0.

Let's start with row 2 (r_2). Remember, we want to perform some linear algebra that will result in the element in this row and the current Pivot Column taking a value of 0.

We could subtract $6*r_1'$ from r_2.

r_2 =	(6.0	0.0	0.0	1.0	-2.0	0.0	1100)
$-6r_1'$ = -6	(1.0	0.0	1/2	0.0	-2.0	0.0	100)
r_2' =	(0.0	0.0	-3.0	1.0	10.0	0.0	500)

The revised row 2 (r_2') will be substituted for the original row 2 (r_2) in the next tableau.

We continue by performing the necessary pivot operations on row 3 & 4 (r_3 & r_4) that will result in the element in these rows and the current Pivot Column taking values of 0.

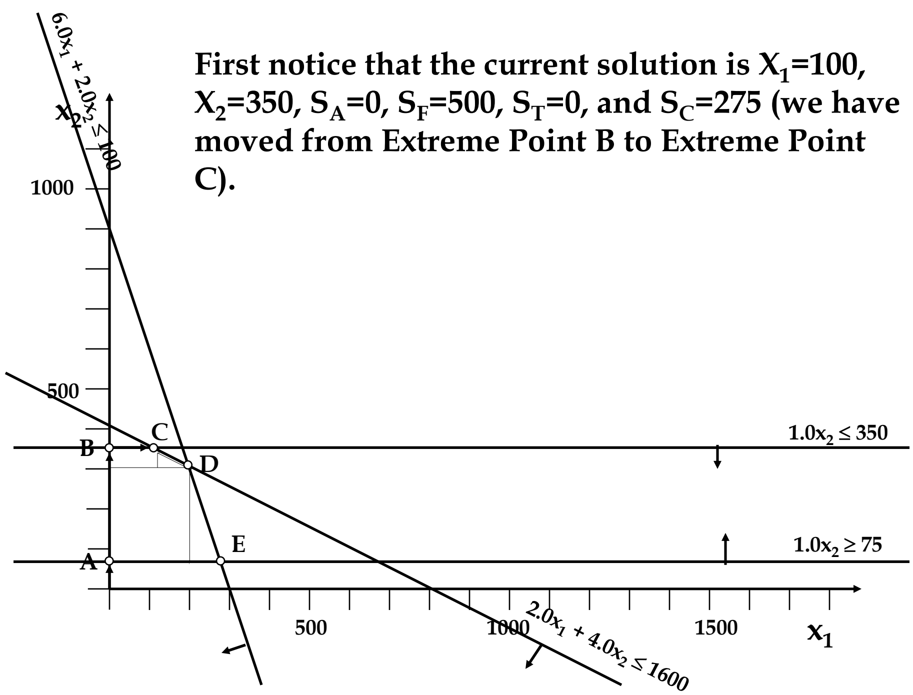
We do not have to do any pivot operations (why?).

Since we have updated all rows in the tableau, we are ready to create a revised tableau.

First we substitute the revised rows r_1', r_2', r_3', and r_4' for the original rows r_1, r_2, r_3, and r_4 (don't forget to update to Basic Mix label and Unit Profit column for r_1!).

Unit Profit		3	8	0	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	S_C	**Solution (π)**	**Exchange Ratios**
3	X_1	1.0	0.0	1/2	0.0	-2.0	0.0	100	
0	S_F	0.0	0.0	-3.0	1.0	10.0	0.0	500	
0	S_C	0.0	0.0	0.0	0.0	1.0	1.0	275	
8	X_2	0.0	1.0	0.0	0.0	1.0	0.0	350	
	Sac.								
	Imp.								

Now we recalculate the Unit Sacrifice and Unit Improvement Rows.



First notice that the current solution is X_1=100, X_2=350, S_A=0, S_F=500, S_T=0, and S_C=275 (we have moved from Extreme Point B to Extreme Point C).

Once we substitute the revised Unit Sacrifice and Unit Improvement Rows into the new tableau, we can identify the Entering variable.

Unit Profit		3	8	0	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	S_C	Solution (π)	Exchange Ratios
3	X_1	1.0	0.0	1/2	0.0	-2.0	0.0	100	
0	S_F	0.0	0.0	-3.0	1.0	10.0	0.0	500	
0	S_C	0.0	0.0	0.0	0.0	1.0	1.0	275	
8	X_2	0.0	1.0	0.0	0.0	1.0	0.0	350	
	Sac.	3	8	3/2	0	2	0	3100	
	Imp.	0	0	-3/2	0	-2	0	----	

There are no nonbasic variables that would improve the current solution - we have the optimal solution!

The optimal solution is

$$X_1 = 100$$
$$X_2 = 350$$
$$S_A = 0$$
$$S_F = 500$$
$$S_T = 0$$
$$S_C = 275$$
$$\pi = 3100$$

In other words, produce 100 Black & White Sets and 350 Color Sets at a profit of $3,100. There will be 500 unused fabrication hours, no unused assembly labor hours or color television tubes, and the minimum production of Color Sets will be exceeded by 275.

e. One More Example - Suppose our television manufacturer is currently only concerned about costs (and not profits). Further suppose his primary concern is labor costs, which are $5.00 per hour for assembly and $3.00 for fabrication. By an agreement with the labor union, at least 200 assembly hours and 300 fabrication hours must be utilized daily.

Our first step is to formulate the problem. Logically, we would want our total costs for this problem to be as low as possible. Since it takes two assembly hours and four fabrication hours to produce a Black & White set, the total cost to produce one Black & White set is

$$2(\$5.00) + 4(\$3.00) = \$22.00$$

Where x_1 is the number of black & white sets produced
x_2 is the number of color sets produced

Since it takes six assembly hours and two fabrication hours to produce a Color set, the total cost to produce one Color set is

$$6(\$5.00) + 2(\$3.00) = \$36.00$$

So the objective function for this problem is

$$\text{minimize } C = 22.0x_1 + 36.0x_2$$

Now we also know we must use at least 200 assembly hours per day, which represents a limitation on our ability to minimize costs. Thus we have a constraint

$$2.0x_1 + 4.0x_2 \geq 200$$

Similarly for fabrication hours we have

$$6.0x_1 + 2.0x_2 \geq 300$$

Thus the ultimate formulation (ignoring the earlier set of constraints) is:

minimize $C = 22.0x_1 + 36.0x_2$

subject to:
$2.0x_1 + 4.0x_2 \geq 200$ **(min # of assembly hours used)**
$6.0x_1 + 2.0x_2 \geq 300$ **(min # of fabrication hours used)**
$x_1, \quad x_2 \geq \quad 0$ **(nonnegativity)**

Where x_1 **is the number of black & white sets produced**
x_2 **is the number of color sets produced**

Although this could be solved graphically (there *are* only two decision variables) we will use this problem to illustrate use of the simplex algorithm to solve minimization problems.

Incorporating the necessary Artificial Variables into the new Television Problem formulation would result in:

minimize $C = 22.0x_1 + 36.0x_2 + 0S_A + 0S_F + MA_A + MA_F$

subject to:

$$2.0x_1 + 4.0x_2 - S_A + A_A = 200$$
$$6.0x_1 + 2.0x_2 - S_F + A_F = 300$$
$$x_1, \; x_2, \; S_A, \; S_F, \; A_A, \; A_F \geq 0$$

Notice that the objective function coefficients for the artificial variables are +M (an arbitrarily large positive value). Since we are minimizing the objective function, this should ensure that the artificial variable is not part of the final optimal solution.

Our initial basic solution can now be:

$$X_1 = 0$$
$$X_2 = 0$$
$$S_A = 0$$
$$S_F = 0$$
$$A_A = 200$$
$$A_F = 300$$

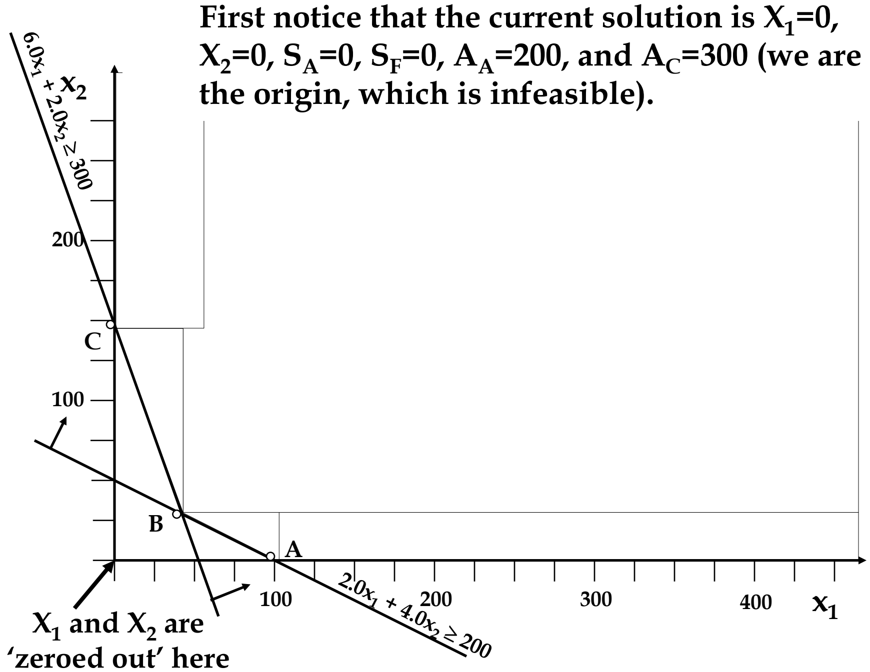
Now let's use simplex to solve the new Television Problem formulation:

The original tableau (with the initial Sacrifice and Improvement rows) looks like this:

Unit Profit	Basic Mix	22 X_1	36 X_2	0 S_A	0 S_F	M A_A	M A_F	Solution (C)	Exchange Ratios
M	A_A	2.0	4.0	-1.0	0.0	1.0	0.0	200	
M	A_F	6.0	2.0	0.0	-1.0	0.0	1.0	300	
	Sac.	8M	6M	-M	-M	M	M	500M	
	Imp.	22-8M	36-6m	M	M	0	0	----	

The current nonbasic variable that will improve the objective function at the greatest rate is X_1 (why is this so?).

First notice that the current solution is $X_1=0$, $X_2=0$, $S_A=0$, $S_F=0$, $A_A=200$, and $A_C=300$ (we are at the origin, which is infeasible).



Now that we have circled the column corresponding to this variable (the *Pivot Column*) we must identify the Exiting Variable

Unit Profit		22	36	0	0	M	M		
	Basic Mix	X_1	X_2	S_A	S_F	A_A	A_F	Solution (C)	Exchange Ratios
M	A_A	2.0	4.0	-1.0	0.0	1.0	0.0	200	200/2
M	A_F	6.0	2.0	0.0	-1.0	0.0	1.0	300	300/6
	Sac.	8M	6M	-M	-M	M	M	500M	
	Imp.	22-8M	36-6m	M	M	0	0	----	

The current basic variable with the *smallest nonnegative* Exchange Ratio is A_F. This is the Exiting Variable.

Now that we have circled the row corresponding to this variable (the *Pivot Row*) we can commence with the Pivot Operations.

We first want to perform some linear algebra that will result in the current Pivot Element taking a value of 1 and all other elements in the Pivot Column to values of 0.

$$r_2' = (1/6)r_2 = (\boxed{1.0} \quad 1/3 \quad 0.0 \quad -1/6 \quad 0.0 \quad 1/6 \quad 50)$$

The revised row 2 (r_2') will be substituted for the original row 2 (r_2) in the next tableau.

Now we want to perform some linear algebra that will result in all other elements in the current Pivot Column taking on values of 0.

We proceed with row 1 (r_1). Remember, we want to perform some linear algebra that will result in the element in this row and the current Pivot Column taking a value of 0.

We could subtract $2*r_2'$ from r_1.

r_1	=	(2.0	4.0	-1.0	0.0	1.0	0.0	200)
$-2r_2'$	= -2	(1.0	1/3	0.0	-1/6	0.0	1/6	50)
r_1'	=	(0.0	10/3	-1.0	1/3	1.0	-1/3	100)

The revised row 1 (r_1') will be substituted for the original row 1 (r_1) in the next tableau.

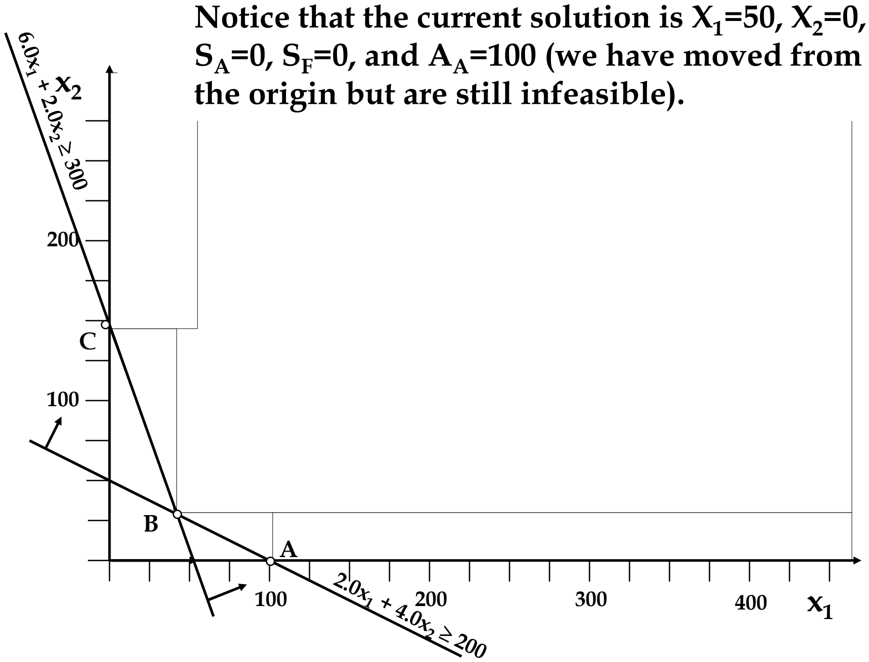
Since we have updated all rows in the tableau, we are ready to create a revised tableau.

First we substitute the revised rows r_1'& r_2' for the original rows r_1 & r_2 (don't forget to update to Basic Mix label and Unit Profit column for r_2 and remember to discard the artificial variable A_F that has exited the basis!).

Unit Profit		22	36	0	0	M		
	Basic Mix	X_1	X_2	S_A	S_F	A_A	Solution (C)	Exchange Ratios
M	A_A	0.0	10/3	-1.0	1/3	1.0	100	
22	X_1	1.0	1/3	0.0	-1/6	0.0	50	
	Sac.							
	Imp.							

Now we recalculate the Unit Sacrifice and Unit Improvement Rows.

Notice that the current solution is X_1=50, X_2=0, S_A=0, S_F=0, and A_A=100 (we have moved from the origin but are still infeasible).



Once we substitute the revised Unit Sacrifice and Unit Improvement Rows into the new tableau, we can identify the Entering variable.

Unit Profit		22	36	0	0	M		
	Basic Mix	X_1	X_2	S_A	S_F	A_A	Solution (C)	Exchange Ratios
M	A_A	0.0	10/3	-1.0	1/3	1.0	100	
22	X_1	1.0	1/3	0.0	-1/6	0.0	50	
	Sac.	22	$\frac{10M+22}{3}$	-M	$\frac{2M-22}{6}$	M	100M+ 1100	
	Imp.	0	$\frac{86-10M}{3}$	M	$\frac{22-2M}{6}$	0	----	

X_2 will enter the basic solution on the next iteration.

Now that we have circled the column corresponding to this variable (the *Pivot Column*) we must identify the Exiting Variable

Unit Profit		22	36	0	0	M		
	Basic Mix	X_1	X_2	S_A	S_F	A_A	Solution (C)	Exchange Ratios
M	A_A	0.0	10/3	-1.0	1/3	1.0	100	$\frac{100}{10/3}$
22	X_1	1.0	1/3	0.0	-1/6	0.0	50	$\frac{50}{1/3}$
	Sac.	22	$\frac{10M+22}{3}$	-M	$\frac{2M-22}{6}$	M	100M+ 1100	
	Imp.	0	$\frac{86-10M}{3}$	M	$\frac{22-2M}{6}$	0	----	

The current basic variable with the *smallest nonnegative* Exchange Ratio is A_A. This is the Exiting Variable.

Now that we have circled the row corresponding to this variable (the *Pivot Row*) we can perform the necessary Pivot Operations.

We first want to perform some linear algebra that will result in the current Pivot Element taking a value of 1 and all other elements in the Pivot Column to values of 0.

$r_1' = (3/10)r_1 = (0.0 \quad \boxed{1.0} \quad -3/10 \quad 1/10 \quad 3/10 \quad 30)$

The revised row 1 (r_1') will be substituted for the original row 1 (r_1) in the next tableau.

We now must perform some linear algebra that will result in all other elements in the current Pivot Column taking on values of 0.

Let's start with row 2 (r_2). Remember, we want to perform some linear algebra that will result in the element in this row and the current Pivot Column taking a value of 0.

We could subtract $1/3*r_1'$ from r_2.

r_2 =	(1.0	1/3	0.0	-1/6	0.0	50)
$-(1/3)r_1'$ =	-1/3(0.0	1.0	-3/10	1/10	3/10	30)
r_2' =	(1.0	0.0	1/10	-1/5	-1/10	40)

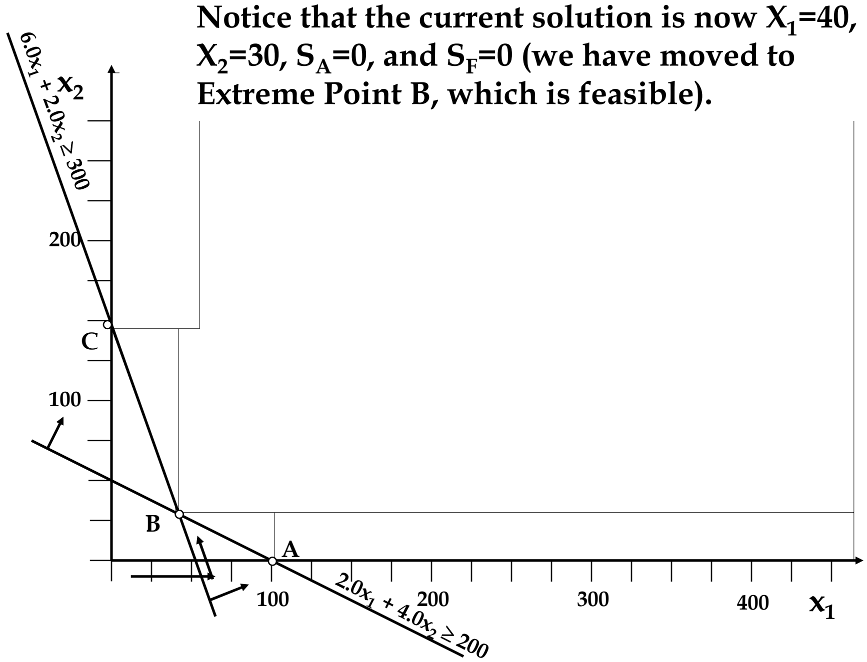
The revised row 2 (r_2') will be substituted for the original row 2 (r_2) in the next tableau.
Since we have updated all rows in the tableau, we are ready to create a revised tableau.

First we substitute the revised rows r_1' & r_2' for the original rows r_1 & r_2 (Don't forget to update to Basic Mix label and Unit Profit column for r_1 and remember to discard the artificial variable A_A that has exited the basis!).

Unit Profit		22	36	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	Solution (C)	Exchange Ratios
36	X_2	0.0	1.0	-3/10	1/10	30	
22	X_1	1.0	0.0	1/10	-1/5	40	
	Sac.						
	Imp.						

Now we recalculate the Unit Sacrifice and Unit Improvement Rows.

Notice that the current solution is now X_1=40, X_2=30, S_A=0, and S_F=0 (we have moved to Extreme Point B, which is feasible).

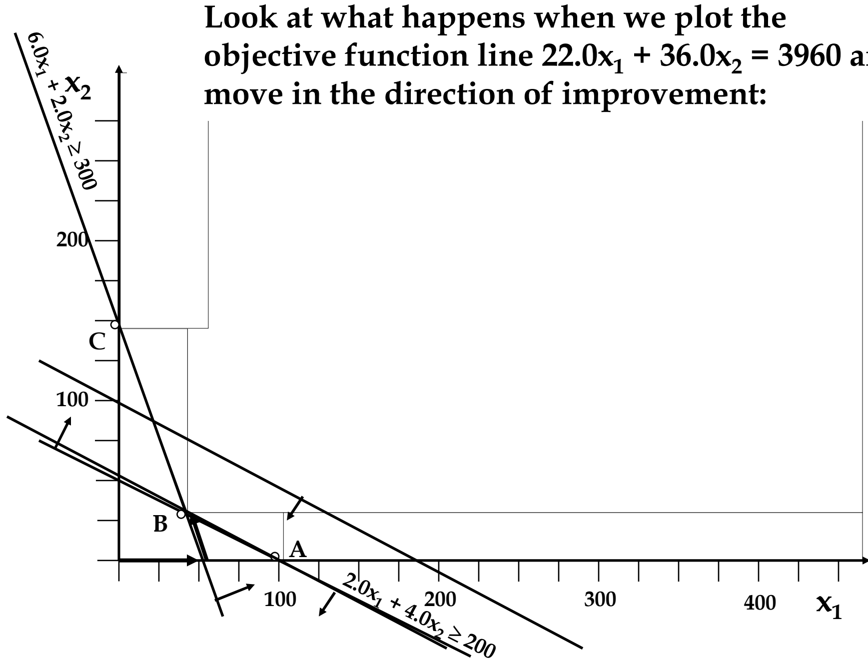


Once we substitute the revised Unit Sacrifice and Unit Improvement Rows into the new tableau, we can identify the Entering variable.

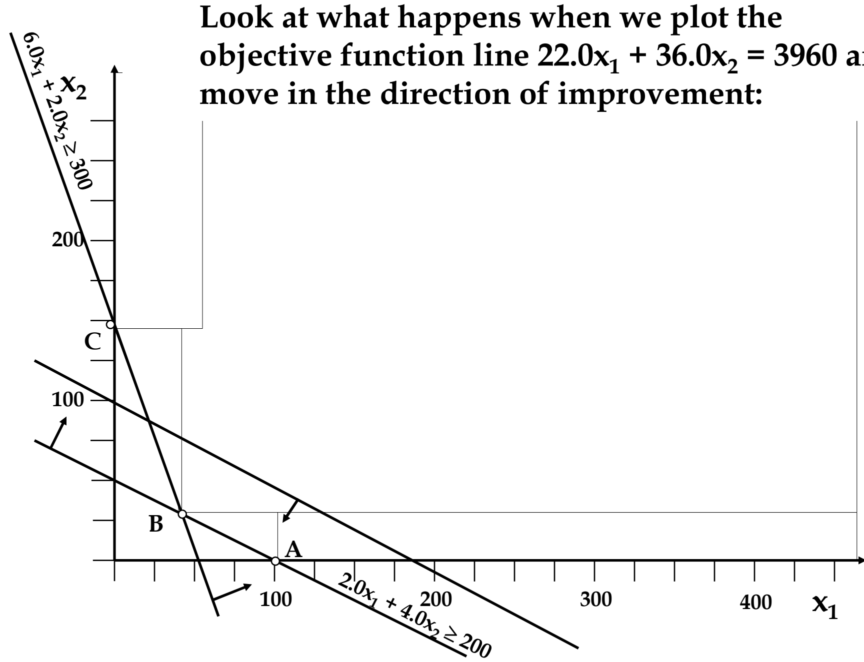
Unit Profit		22	36	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	Solution (C)	Exchange Ratios
36	X_2	0.0	1.0	-3/10	1/10	30	
22	X_1	1.0	0.0	1/10	-1/5	40	
	Sac.	22	36	-86/10	-8/10	1960	
	Imp.	0	0	86/10	8/10	----	

There are no nonbasic variables that would improve the current solution - we have the optimal solution!

Look at what happens when we plot the objective function line $22.0x_1 + 36.0x_2 = 3960$ and move in the direction of improvement:



Look at what happens when we plot the objective function line $22.0x_1 + 36.0x_2 = 3960$ and move in the direction of improvement:



The optimal solution is

$X_1 = 40$
$X_2 = 30$
$S_A = 0$
$S_F = 0$
$C = 1960$

In other words, produce 40 Black & White Sets and 30 Color Sets at a cost of $1,960. You will use no fabrication hours or assembly labor hours beyond the agreed upon minimums.

e. Special Problems in Simplex Tabeaus

- Multiple (Alternate) Optimal Solutions - more than one extreme point provides an optimal value of the objective function. An example of a final simplex tableau *from a Minimization Problem* exhibiting Alternate Optimal Solutions is:

Unit Profit		33	11	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	Solution (C)	Exchange Ratios
11	X_2	0.0	1.0	-3/10	1/10	30	
33	X_1	1.0	0.0	1/10	-1/5	40	
	Sac.	33	11	0	-77/10	1650	
	Imp.	0	0	0	77/10	----	

Notice that surplus variable S_A could enter the basis without hurting (or helping) the optimal solution of this minimization problem!

- Infeasibility - no set of values for the decision variables is feasible. An example of a simplex tableau *from a Minimization Problem* exhibiting Infeasibility is:

Unit Profit		22	36	0	0	M		
	Basic Mix	X_1	X_2	S_A	S_F	A_A	Solution (C)	Exchange Ratios
M	A_A	0.0	-10/3	-1.0	-1/3	1.0	100	
22	X_1	1.0	1/3	0.0	-1/6	0.0	50	
	Sac.	22	$\frac{22-10M}{3}$	-M	$\frac{-2M-22}{6}$	M	100M+ 1000	
	Imp.	0	$\frac{10M+86}{3}$	M	$\frac{2M+22}{6}$	0	----	

Notice that there are no viable candidates to enter the basis, yet an artificial variable (A_A) is still part of the basic solution!

- **Degeneracy - occurs when at least one basic variable takes a value of zero in the final solution. An example of a simplex tableau *from a Minimization Problem* exhibiting Degeneracy is:**

Unit Profit	Basic Mix	33 X_1	11 X_2	0 S_A	0 S_F	Solution (C)	Exchange Ratios
11	X_2	0.0	1.0	-3/10	1/10	0	
33	X_1	1.0	0.0	1/10	-1/5	40	
	Sac.	33	11	0	-77/10	1650	
	Imp.	0	0	0	77/10	----	

Notice that X_1 is in the basis of the final solution, yet has a value of zero!

An early warning of Degeneracy occurs in the iteration prior to the final tableau, where a tie for the entering variable occurs.

Note that Degeneracy will *generally* not create problems. However, if degeneracy occurs at a suboptimal step, cycling (alternating continuously between two or more nonoptimal solutions in succeeding iterations) may occur.

If cycling does occur, return to the tableau where the tie occurred and chose the other entering variable (i.e., the entering variable that you did not select originally) and proceed with your iterations.

- Unboundedness - exists when no exiting variable (basis variable with a positive exchange coefficient) corresponding to an entering variable with a positive exchange exists. An example of a simplex tableau *from a Maximization Problem* exhibiting unboundedness is:

Unit Profit		3	8	0	0	0	0		
	Basic Mix	X_1	X_2	S_A	S_F	S_T	S_C	Solution (π)	Exchange Ratios
0	S_A	-2.0	0.0	1.0	0.0	-4.0	0.0	200	200/-2
0	S_F	-6.0	0.0	0.0	1.0	-2.0	0.0	1100	1100/-6
0	S_C	0.0	0.0	0.0	0.0	1.0	1.0	275	275/0
8	X_2	0.0	1.0	0.0	0.0	1.0	0.0	350	350/0
	Sac.	0	8	0	0	8	0	2800	
	Imp.	3	0	0	0	-8	0	----	

4. Using the Computer to Solve LP Problems

a. Programming Languages

- Fortran
- C ++

b. Dedicated Software

- Lindo
- CPlex

c. Spreadsheet Software

- Frontline Systems' Solver (available in Excel)

Steps in Using Excel Solver

Step 1: **Select and Label *Changing Variable Cells* (cells that will contain the values of individual decision variables**

Step 2: **In the *Set (Target) Cell* write the Objective Function as a cell function of the Changing Variable Cells**

Step 3: **In *Subject to the Constraint Cells* write each Constraint as a cell function of the Adjustable Cells. Also enter the corresponding Right-Hand Sides in individual cells**

Step 4: **Under *Tools* select the *Solver* option - when the *Solver Parameters dialog box* appears**

- **Enter the cell address of the Target Cell in the *Set Cell* box**
- **Select the *MAX* or *MIN* option**
- **Enter the cell addresses of the Changing Cells in the *By Changing Variables Cells* box**
- **Use the ADD option and enter the Changing Cell in the *Cell Reference box*, the Right Hand Side in the *Constraint box*, and selecting the relationship between the two sides**
- **Select the *Assume Linear Model box***
- **Select the *Assume Non-Negative box***
- **Select *OK***
- **Choose *Solve***

Step 5: When Solver Results dialog box appears

- **Select Keep Solver Solution**
- **Choose *OK To Produce Optimal Solution Output***

Example – the Original Television Problem

maximize $\pi = 3.0x_1 + 8.0x_2$

subject to:

$2.0x_1 + 4.0x_2 \leq 1600$ **(# of assembly hours available)**

$6.0x_1 + 2.0x_2 \leq 1800$ **(# of fabrication hours available)**

$1.0x_2 \leq 350$ **(# of available color tubes)**

$1.0x_2 \geq 75$ **(minimum # of color sets)**

$x_1, \quad x_2 \geq 0$ **(nonnegativity)**

Where x_1 **is the number of black & white sets produced**

x_2 **is the number of color sets produced**