IX. Decision Theory

Techniques used to find optimal solutions in situations where a decision maker is faced with several alternatives (Actions) and an uncertain or risk-filled future (Events or States of Nature). The techniques can utilize knowledge of the possible actions that the decision maker can take, probabilities that the events will occur, and the outcome (payoff) associated with each combination of action and event.

A. Basic Definitions

Act/Action - Choice available to the decision maker.

Outcome/Payoff - The result of an act.

Event/State of Nature - Uncontrollable occurrence that will affect the payoff associated with an action.

Payoff Table - Tabular display of actions, events, and associated outcomes. As an example:

Payoff Table

	States of Nature		
Decision Alternatives	Sales Drop (E_1)	Sales Constant (E_2)	Sales Increase (E_3)
Purchase New Equipment (A_1)	10	60	85
Not Purchase New Equipment (A_2)	30	40	60

B. Approaches to Decision Making

1. **Decision Theoretic approaches for situations in which probabilities associated with the events are not available:**
   - **Optimistic (Maximax) Decision Making: Choose the act associated with the largest possible outcome.**
   - **Conservative (Maximin) Decision Making: Choose the act associated with the largest minimum outcome.**
   - **Minimax Regret Decision Making: Calculate the regret table by**

   a) **finding the maximum payoff associated with each event;**

   b) **subtracting every outcome from a given event from the corresponding maximum payoff; then**

   c) **choosing the act associated with the smallest maximum regret.**

Example - Find the Optimistic, Conservative, and Minimax Regret acts associated with the following payoff table.

Payoff Table

	Events			
Decision Alternatives	Sales Drop	Sales Constant	Sales Increase	Maximum Payoffs
Purchase New Equipment	10	60	85	85
Do Not Purchase New Equipment	30	40	60	60

Optimistic (Maximax) Decision - Purchase New Equipment (maximum payoff of 85);

Example - Find the Optimistic, Conservative, and Minimax Regret acts associated with the following payoff table.

Payoff Table

Decision Alternatives	Events: Sales Drop	Sales Constant	Sales Increase	Minimum Payoffs
Purchase New Equipment	10	60	85	10
Do Not Purchase New Equipment	30	40	60	30

Conservative (Maximin) Decision - Do not Purchase New Equipment (largest minimum payoff - 30);

Example - Find the Optimistic, Conservative, and Minimax Regret acts associated with the following payoff table.

Regret Table

Decision Alternatives	Events: Sales Drop	Sales Constant	Sales Increase	Maximum Regret
Purchase New Equipment	20	0	0	20
Do Not Purchase New Equipment	0	20	25	25

Minimax Regret Decision Making - Purchase New Equipment (maximum regret of 25).

2. Decision Theoretic approaches for situations in which probabilities associated with the events are available:

- **Maximum Likelihood Decision Making - Choose the optimal decision alternative associated with the most likely event.**
- **Maximum Expected Value Decision Making (Bayes' Decision Rule) - Choose the decision alternative associated with the best probability-weighted payoff.**
- **Insufficient Reason Decision Making – assume all events are equally likely and choose the decision alternative associated with the best probability-weighted payoff.**

Consider the previous problem revised to include probabilities for the three states of nature.

Payoff Table

	Events		
Decision Alternatives	Sales Drop p=0.5	Sales Constant p=0.3	Sales Increase p=0.2
Purchase New Equipment	10	60	85
Do Not Purchase New Equipment	30	40	60

The Maximum Likelihood Decision would be to *not purchase the new equipment* (the maximum likelihood event is *Sales Drop*, and best payoff associated with this event is 30 for not purchasing the new equipment).

The expected values of the two potential decisions are

E(Purchase New Equipment) = .5(10) + .3(60) + .2(85) = 40
E(Not Purchase New Equipment) = .5(30) + .3(40) + .2(60) = 39

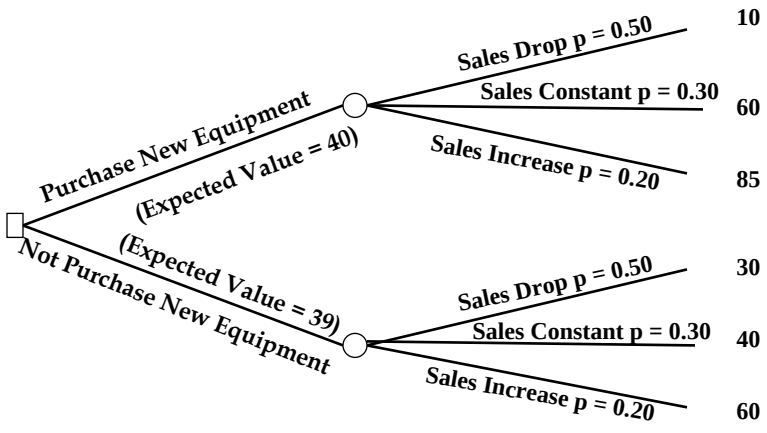
Payoff Table

	Events		
Decision Alternatives	Sales Drop p=0.5	Sales Constant p=0.3	Sales Increase p=0.2
Purchase New Equipment	10	60	85
Do Not Purchase New Equipment	30	40	60

The Maximum Expected Value Decision would be to *purchase the new equipment* (the maximum expected value is 40, which is associated with purchasing the new equipment).

So the Maximum Expected Value decision is to Purchase New Equipment (expected payoff of 40).

This can also be seen with a decision tree:



If we assume all three events are equally likely, The expected values of the two potential decisions are

E(Purchase Equipment) = .33(10) + .33(60) + .33(85) = 51.67
E(Not Purchase Equipment) = .33(30) + .33(40) + .33(60) = 43.33

Payoff Table

	Events		
Decision Alternatives	Sales Drop p=0.33	Sales Constant p=0.33	Sales Increase p=0.33
Purchase New Equipment	10	60	85
Do Not Purchase New Equipment	30	40	60

The Insufficient Reason Decision is to *purchase the new equipment*.

3. Decision Theoretic approaches using *Certainty Equivalents*

Certainty Equivalent – payoff amount for which the decision maker would be willing to relinquish participation in an uncertain situation (how much would you require to sell a ticket in a $1,000,000,000 lottery?).

Risk Premium – difference between the expected value and the certainty equivalent for an act:

Risk Premium = expected value - certainty equivalent

Note that often times risk premiums are estimated and subtracted from expected values to derive estimates certainty equivalents

- **Greatest Certainty Equivalent – Assign certainty equivalents to each act, then select the act with the greatest assigned certainty equivalent.**

C. Additional Information for Decision Making

Regret/Opportunity Loss – difference between the payoff for a chosen act and the best payoff that could have been achieved. Can be put into an Opportunity Loss/Regret Table:

Regret Table

Decision Alternatives	Events: Sales Drop	Sales Constant	Sales Increase	Maximum Regret
Purchase New Equipment	20	0	0	20
Do Not Purchase New Equipment	0	20	25	25

1. **Expected Regret/Opportunity Loss – probability weighted Opportunity Loss/Regret. For our problem:**

Regret Table

Decision Alternatives	Events: Sales Drop p=0.5	Sales Constant p=0.3	Sales Increase p=0.2	Maximum Regret
Purchase New Equipment	20	0	0	20
Do Not Purchase New Equipment	0	20	25	25

The Expected Regret/Opportunity Loss is:

0.50(20) + 0.30(0) + 0.20(0) = 10 for *Purchase Equipment*

0.50(0) + 0.30(20) + 0.20(25) = 11 for *Don't Purchase Equipment*

Our decision under Minimum Expected Regret/ Opportunity Loss is Purchase New Equipment (note that this is *always* consistent with Bayes' Criteria)

Perfect Information – absolute certain knowledge of when each event will occur.

2. **Expected Payoff Under Certainty (EPUC) – the value of the decisions that will be made if the decision maker can obtain absolute certain knowledge of when each event will occur.**

   Consider our previous example:

Payoff Table

	Events		
Decision Alternatives	Sales Drop p=0.5	Sales Constant p=0.3	Sales Increase p=0.2
Purchase New Equipment	10	60	85
Do Not Purchase New Equipment	30	40	60
Maximum	30	60	85

The Expected Payoff Under Certainty (EPUC) is:

$$EPUC = 0.50(30) + 0.30(60) + 0.20(85) = 50$$

3. **Expected Value of Perfect Information (EVPI) – the difference between the Expected Payoff Under Certainty and the Expected Value (with no additional information)**

EVPI = EPUC – Maximum Payoff under Expected Value (Expected Value Without Perfect Information or EVwoPI)

For our example we have

$$EVPI = 50 - 40 = 10$$

Any information we can obtain will be worth *at most* $10.00.

Note that EVPI and Minimum Regret/Opportunity Loss are equivalent!

Perfect Information – absolute certain knowledge of when each event will occur.

2. **Expected Value of Perfect Information (EVPI) – the value of the decisions that will be made if the decision maker can obtain absolute certain knowledge of when each event will occur.**

Consider our previous example:

Payoff Table

	Events		
Decision Alternatives	Sales Drop p=0.5	Sales Constant p=0.3	Sales Increase p=0.2
Purchase New Equipment	10	60	85
Do Not Purchase New Equipment	30	40	60
Maximum	30	60	85

The Expected Value of Perfect Information is:

$$EPUC = 0.50(30) + 0.30(60) + 0.20(85) = 50$$